

Teleportation using continuous variable quantum cloning machine

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Abstract

We show that an unknown quantum state in phase space can be teleported via three-mode entanglement generated by continuous variable quantum cloning machine (transformation). Further, proceeding with our teleportation protocol we are able to improve the fidelity of teleportation obtained by Loock et.al. [Phys.Rev.Lett. 84, 3482(2000)]. Also we study here the entanglement between the two output copies from cloning machine.

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Quantum information theory was initiated with the discrete quantum variables but now a days it has been extended to the domain of continuous quantum variables. The possible reason for this may be the continuous spectrum systems are experimentally simpler to manipulate than their discrete counterparts in order to process quantum information. The second reason to switch over from the discrete quantum variables to the continuous variables is the *Unconditionalness*, which is one of the valuable feature of quantum optical implementations based upon continuous variables. But this valuable feature give a price in terms of the quality of the entanglement. The entanglement and the entanglement-based quantum protocol is always imperfect and the degree of imperfection depends on the amount of squeezing of the laser light involved. Good performance of the entanglement-based quantum protocol can be achieved for large squeezing (about 10 dB [24]) which is technologically demanding also. In this letter, our aim is two fold: First, we consider the continuous variable cloning transformation and study the entanglement between the two output copies by covariance matrix approach. Second, We will show that the three-mode entanglement (two copy mode and one ancilla mode) generated by cloning procedure can be used to teleport a single arbitrary mode with optimum fidelity $F^{opt} = \left[\frac{1+3e^{-4r}+2e^{-6r}+2e^{-2r}}{1+2e^{-4r}} \right]^{\frac{-1}{2}}$. We will observe that

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the optimum fidelity F^{opt} obtained by our protocol is greater than or equal to the optimum fidelity $F'_{opt} = [(1+e^{-2r})(1+\frac{3}{(2e^{2r}+e^{-2r})})]^{\frac{-1}{2}}$ obtained by Loock and Braunstein [23].

Quantum cloning: An arbitrary quantum state cannot be cloned perfectly but it does not rule out the approximate cloning [1, 2]. There are numerous works in the literature on quantum cloning in discrete variables as well as in continuous variables [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 20, 21, 22, 37]. The experimental realization of the Gaussian cloning machine with three NOPAs has been discussed in [19] but this cloning device is approximate and the desired cloning transformation is achieved only in the limit of infinite squeezing. Later this problem was solved by Fiurasek [16] and almost in the same time by Braunstein et.al. [17]. They showed that such problems are avoided if sequence of beam splitters and a phase-insensitive linear amplifier are used to realize the cloning transformation.

The $1 \rightarrow 2$ continuous variable cloning transformation can be realized by two canonical transformations (i) two mode Bogoliubov transformation for phase-insensitive amplifier with gain 2 applied on the input mode \hat{a}_0 and the ancilla mode \hat{a}_z and (ii) phase free 50:50 beam splitter operation operated on the output mode \hat{a}'_0 from the amplifier and on the blank mode \hat{a}_1

$$\begin{aligned}\hat{a}'_0 &= \sqrt{2}\hat{a}_0 + \hat{a}_z^\dagger, & \hat{a}'_z &= \hat{a}_0^\dagger + \sqrt{2}\hat{a}_z, \\ \hat{a}''_0 &= \frac{(\hat{a}'_0 + \hat{a}_1)}{\sqrt{2}}, & \hat{a}''_1 &= \frac{(\hat{a}'_0 - \hat{a}_1)}{\sqrt{2}}\end{aligned}\quad (1)$$

where $\hat{a}_k = \frac{(\hat{x}_k + i\hat{p}_k)}{\sqrt{2}}$ and $\hat{a}_k^\dagger = \frac{(\hat{x}_k - i\hat{p}_k)}{\sqrt{2}}$ denote the annihilation and creation operators for mode k. \hat{a}_0 is the input mode, \hat{a}_1 is the blank mode on which the information is to be copied and \hat{a}_z is the ancilla mode.

In terms of position and momentum operators, the transformation for any input mode reduces to

$$\begin{aligned}\hat{x}''_0 &= \hat{x}_0 + \frac{\hat{x}_1}{\sqrt{2}} + \frac{\hat{x}_z}{\sqrt{2}}, & \hat{p}''_0 &= \hat{p}_0 + \frac{\hat{p}_1}{\sqrt{2}} - \frac{\hat{p}_z}{\sqrt{2}}, \\ \hat{x}''_1 &= \hat{x}_0 - \frac{\hat{x}_1}{\sqrt{2}} + \frac{\hat{x}_z}{\sqrt{2}}, & \hat{p}''_1 &= \hat{p}_0 - \frac{\hat{p}_1}{\sqrt{2}} - \frac{\hat{p}_z}{\sqrt{2}}, \\ \hat{x}'_z &= \hat{x}_0 + \sqrt{2}\hat{x}_z, & \hat{p}'_z &= -\hat{p}_0 + \sqrt{2}\hat{p}_z\end{aligned}\quad (2)$$

If we assume, without loss of generality, that the input and blank mode are squeezed in 'x' with squeezing parameter r_0 and r_1 respectively and the ancilla mode is squeezed in 'p' with squeezing parameter r_z then the cloning transformation for single mode squeezed state input in the Heisenberg picture is given by

$$\begin{aligned}\hat{x}''_0 &= \hat{x}_0^{(0)}e^{-r_0} + \frac{\hat{x}_1^{(0)}e^{-r_1}}{\sqrt{2}} + \frac{\hat{x}_z^{(0)}e^{r_z}}{\sqrt{2}}, & \hat{p}''_0 &= \hat{p}_0^{(0)}e^{r_0} + \frac{\hat{p}_1^{(0)}e^{r_1}}{\sqrt{2}} - \frac{\hat{p}_z^{(0)}e^{-r_z}}{\sqrt{2}}, \\ \hat{x}''_1 &= \hat{x}_0^{(0)}e^{-r_0} - \frac{\hat{x}_1^{(0)}e^{-r_1}}{\sqrt{2}} + \frac{\hat{x}_z^{(0)}e^{r_z}}{\sqrt{2}}, & \hat{p}''_1 &= \hat{p}_0^{(0)}e^{r_0} - \frac{\hat{p}_1^{(0)}e^{r_1}}{\sqrt{2}} - \frac{\hat{p}_z^{(0)}e^{-r_z}}{\sqrt{2}},\end{aligned}$$

$$\hat{x}'_z = \hat{x}_0^{(0)} e^{-r_0} + \sqrt{2} \hat{x}_z^{(0)} e^{r_z}, \quad \hat{p}'_z = -\hat{p}_0^{(0)} e^{r_0} + \sqrt{2} \hat{p}_z^{(0)} e^{-r_z} \quad (3)$$

After tracing out the ancilla mode and putting $r_z = 0$, the second moment correlation matrix (Covariance matrix) of the two clone mode is given by

$$\sigma_{12} = \begin{pmatrix} s & 0 & t & 0 \\ 0 & u & 0 & v \\ t & 0 & s & 0 \\ 0 & v & 0 & u \end{pmatrix} \quad (4)$$

where

$$\begin{aligned} s &= \frac{1}{4} \left(e^{-2r_0} + \frac{e^{-2r_1}}{2} + \frac{1}{2} \right) \\ t &= \frac{1}{4} \left(e^{-2r_0} - \frac{e^{-2r_1}}{2} + \frac{1}{2} \right) \\ u &= \frac{1}{4} \left(e^{2r_0} + \frac{e^{2r_1}}{2} + \frac{1}{2} \right) \\ v &= \frac{1}{4} \left(e^{2r_0} - \frac{e^{2r_1}}{2} + \frac{1}{2} \right) \end{aligned} \quad (5)$$

The fidelity of the two clones can be evaluated through the relation [36]

$$F^{clone} = \frac{1}{\sqrt{\text{Det}[\sigma_{in} + \sigma_{out}] + \delta} - \sqrt{\delta}} \quad (6)$$

where $\delta = 4(\text{Det}[\sigma_{in}] - 1/4)(\text{Det}[\sigma_{out}] - 1/4)$.

The input state and the two output cloned states are described by the covariance matrices $\sigma_{in} = \begin{pmatrix} \frac{e^{-2r_0}}{4} & 0 \\ 0 & \frac{e^{2r_0}}{4} \end{pmatrix}$ and $\sigma_1^{clone} = \sigma_2^{clone} = \begin{pmatrix} s & 0 \\ 0 & u \end{pmatrix}$.

After some simple algebra, the fidelity of cloning is calculated to be

$$F^{clone} = \frac{16}{\sqrt{434 + 71(e^{2r} + e^{-2r})} - \sqrt{18 - 9(e^{2r} + e^{-2r})}} \quad (7)$$

where $r_0 = r_1 = r$.

If the single mode coherent state (*i.e.* the mode with squeezing parameter $r = 0$) is given to the cloning machine as input then the fidelity of cloning is found to be $\frac{2}{3}$.

Now we investigate the qualification and quantification of entanglement exist (if any) between the two clone modes. The positivity of the partially transposed state (PPT) criterion is necessary and sufficient for the separability of two-mode Gaussian states [25, 26]. Further, J.Laurat et.al. [27] showed that the PPT criterion and an inequality satisfied by the smallest symplectic eigen value $\tilde{\nu}_-$ of the partially transposed state are equivalent. Therefore, the two mode Gaussian state is entangled if the smallest symplectic eigenvalue $\tilde{\nu}_-$ of the partially transposed state is less than one *i.e.* $\tilde{\nu}_- < 1$

provided that the eigen values of the covariance matrix is positive. The eigen values of the covariance matrix σ_{12} is given by $\lambda_1 = s + t, \lambda_2 = s - t, \lambda_3 = u + v, \lambda_4 = u - v$. We can now easily verify that all the four eigen values of σ_{12} are positive for any non-negative squeezing parameter and hence the state represented by the covariance matrix σ_{12} is physical. In this work, we use the logarithmic negativity E_N defined by $E_N = \max[0, -\log_2(\tilde{\nu}_-)]$ to quantify the entanglement.

The smallest symplectic eigenvalues $\tilde{\nu}_-$ of the partial transposed state $\tilde{\sigma}_{12}$ read

$$\begin{aligned}\tilde{\nu}_- &= \sqrt{\frac{\tilde{\Delta}(\sigma_{12}) - \sqrt{\tilde{\Delta}(\sigma_{12})^2 - 4\text{Det}\sigma_{12}}}{2}} \\ &= \sqrt{(s+t)(u-v)} = \frac{1}{4}\sqrt{(2e^{-2r_0} + 1)e^{2r_1}}\end{aligned}\quad (8)$$

where $\tilde{\Delta}(\sigma_{12}) = \Delta(\tilde{\sigma}_{12}) = 2(su - tv)$ and $\text{Det}(\sigma_{12}) = (s^2 - t^2)(u^2 - v^2)$.

Now we consider two cases:

Case-I: If $r_1 = 0$ and $r_0 \rightarrow \infty$ then $\tilde{\nu}_- = \frac{1}{4} < 1$. Therefore, if the blank mode is prepared in a vacuum mode (coherent state) and for the large squeezing of the input mode, the two clone modes are entangled. The amount of entanglement is given by $E_N = \log_2(4)$.

Case-II: If $r_0 = r_1 = r$ then $\tilde{\nu}_- = \frac{\sqrt{2+e^{2r}}}{4}$. The amount of entanglement is given by $E_N = -\log_2(\frac{\sqrt{2+e^{2r}}}{4})$. From the figure (1), it is clear that the two clone modes are entangled only when the squeezing parameter r takes the value lesser than 1.32.

Quantum teleportation: The idea of teleportation was first introduced by Bennett et.al. [28] and this ingenious concept is about the transmission and reconstruction of the state of a quantum system over arbitrary distances [29, 30]. The teleportation of continuous quantum variables such as position and momentum of a particle, as first proposed by Vaidman [31] relies on the entanglement of the states in the original EPR paradox [32]. In quantum optical terms, the observables analogous to the two conjugate variables position and momentum of a particle are the quadratures of a single mode of the electromagnetic field. Continuous variable quantum teleportation of arbitrary coherent states has been realized experimentally with bipartite entanglement built from two single-mode squeezed vacuum states combined at a beam splitter [33].

Loock and Braunstein [23] in their work showed that the twice application of beam splitter operations known as tritter [34] defined by $\hat{T}_{123} \equiv \hat{B}_{23}(\pi/4)\hat{B}_{12}(\cos^{-1}\frac{1}{\sqrt{3}})$ to a zero-momentum eigenstate in mode 1 and a pair of zero-position eigenstates in modes 2 and 3 yields a GHZ-like state. Further they showed that using the generated tripartite entanglement as a quantum channel and with the help of classical communications, one can teleport an unknown quantum state with optimum fidelity $F'_{opt} = [(1 + e^{-2r})(1 + \frac{3}{(2e^{2r} + e^{-2r})})]^{-\frac{1}{2}}$. They found out that the perfect teleportation can be achieved for infinite squeezing while if the squeezing parameters takes the value zero then the optimal fidelity for teleportation achieved is 0.5. In this letter, we will show that the three mode entanglement generated using the cloning transformation can be

used to modify the fidelity of teleportation F'_{opt} .

Now we are in a position to discuss the teleportation of an unknown quantum state through cloning procedure. Our teleportation protocol can be illustrated in the following way: Firstly, Alice use the cloning machine (1) to make two copies of a single mode squeezed state \hat{a}_0 . After the completion of copying procedure, the entanglement between the two copy modes \hat{a}_0'' and \hat{a}_1'' and the ancilla mode \hat{a}_z' is generated. Alice then keeps one copy mode say \hat{a}_0'' and ancilla mode \hat{a}_z' with herself and send another copy mode \hat{a}_1'' to her distant partner Bob. We want to use these three-mode (two copy mode and one ancilla mode) entanglement as a quantum resource to teleport an unknown quantum state. Alice now possesses two modes (\hat{a}_0'' and \hat{a}_z') and Bob holds the mode (\hat{a}_1''). To teleport an arbitrary input mode described in the phase-space $\hat{a}^{in} = (\hat{x}_{in}, \hat{p}_{in})$, Alice combines this input mode with the copy mode $\hat{a}_0'' = (\hat{x}_0'', \hat{p}_0'')$: $\hat{x}_m = \frac{\hat{x}_{in} - \hat{x}_0''}{\sqrt{2}}$, $\hat{p}_n = \frac{\hat{p}_{in} + \hat{p}_0''}{\sqrt{2}}$. Bob's mode $\hat{a}_1'' = (\hat{x}_1'', \hat{p}_1'')$ and Alice's ancilla mode $\hat{a}_z' = (\hat{x}_z', \hat{p}_z')$ read as

$$\begin{aligned}\hat{x}_1'' &= \hat{x}_{in} - (\hat{x}_0'' - \hat{x}_1'') - \sqrt{2}\hat{x}_m \\ \hat{p}_1'' &= \hat{p}_{in} + (\hat{p}_0'' + \hat{p}_1'' + g^{(3)}\hat{p}_z') - \sqrt{2}\hat{p}_n - g^{(3)}\hat{p}_z' \\ \hat{x}_z' &= \hat{x}_{in} - (\hat{x}_0'' - \hat{x}_z') - \sqrt{2}\hat{x}_m \\ \hat{p}_z' &= \hat{p}_{in} - (\hat{p}_0'' + g^{(3)}\hat{p}_1'' + \hat{p}_z') - \sqrt{2}\hat{p}_n - g^{(3)}\hat{p}_1''\end{aligned}\tag{9}$$

Where $g^{(3)}$ denotes the gain and its value which maximizes the fidelity of teleportation will be determined later.

Alice then make measurements on the modes (\hat{x}_m, \hat{p}_n) and (\hat{x}_z', \hat{p}_z') and thereafter send the measured values (x_m, p_n) for (\hat{x}_m, \hat{p}_n) and (x_z', p_z') for (\hat{x}_z', \hat{p}_z') to Bob through a classical channel. Then Bob displace his mode according to the Alice's measured values. After displacement the teleported mode in the Bob's side becomes

$$\begin{aligned}\hat{x}_{tel} &= \hat{x}_{in} - \sqrt{2}\hat{x}_1^{(0)}e^{-r_1} \\ \hat{p}_{tel} &= \hat{p}_{in} + [(2 - g^{(3)})\hat{p}_0^{(0)}e^{r_0} + \sqrt{2}(g^{(3)} - 1)\hat{p}_2^{(0)}e^{-r_z}]\end{aligned}\tag{10}$$

Now to see the efficiency of our teleportation scheme, we have to calculate the fidelity of teleportation. It is given by the formula [35]

$$\begin{aligned}F &= \pi Q_{tel}(x_{in} + ip_{in}) \\ &= \frac{1}{2\sqrt{\sigma_x\sigma_p}} \exp[-(1 - g)^2(\frac{x_{in}^2}{2\sigma_x} + \frac{p_{in}^2}{2\sigma_p})]\end{aligned}\tag{11}$$

where g is the gain and σ_x and σ_p are the variances of the Q function of the teleported mode for the corresponding quadratures.

For $g=1$, it becomes

$$F = \frac{1}{2\sqrt{\sigma_x\sigma_p}}\tag{12}$$

When $r_0 = r_1 = r_z = r$, the teleportation fidelity attains its optimum value for $g^{(3)} = \frac{2(e^{2r}+e^{-2r})}{(e^{2r}+2e^{-2r})}$. Therefore, the optimum fidelity is given by

$$F^{opt} = \left[\frac{1 + 3e^{-4r} + 2e^{-6r} + 2e^{-2r}}{1 + 2e^{-4r}} \right]^{-\frac{1}{2}} \quad (13)$$

(i) If $r \rightarrow \infty$, then $g^{(3)} = 2$ and $F^{opt} = 1$. Therefore, for infinite squeezing the optimum teleportation fidelity goes to unity and hence perfect teleportation is achieved.

(ii) If $r = 0$ then $g^{(3)} = \frac{4}{3}$ and $F^{opt} = \frac{\sqrt{3}}{2\sqrt{2}} > \frac{1}{2}$. Note that in this case the optimum fidelity F^{opt} overtakes the optimum fidelity F'_{opt} obtained by Loock et.al. In figure(2), the solid line represents the optimum fidelity F^{opt} obtained in our protocol and the dotted line represents the optimum fidelity F'_{opt} obtained in Loock's et.al. protocol. Therefore, with the help of figure (2) we showed that the line of the optimum fidelity F^{opt} never go down the line of the optimum fidelity F'_{opt} .

In summary, we showed that one can improve the fidelity of teleportation of a single mode squeezed state to a certain extent if continuous variable cloning machine is used to prepare three-mode entangled state instead of tritter.

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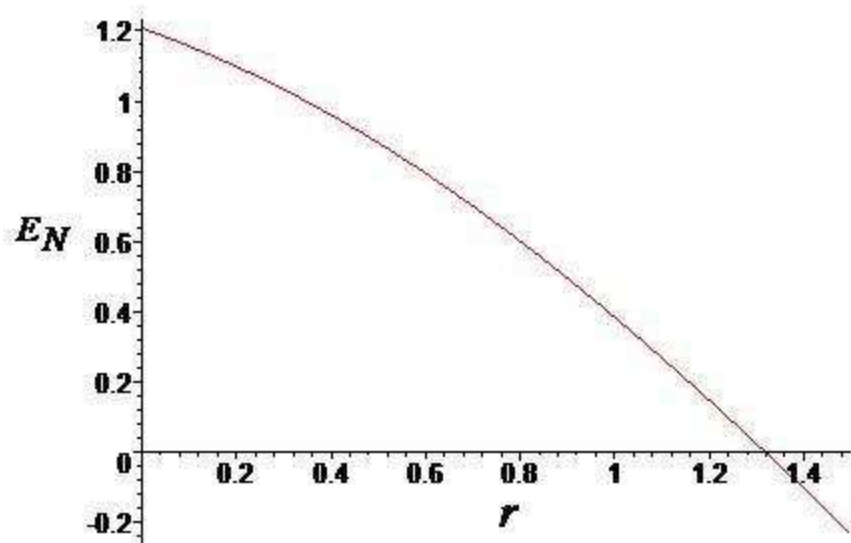


Fig.1

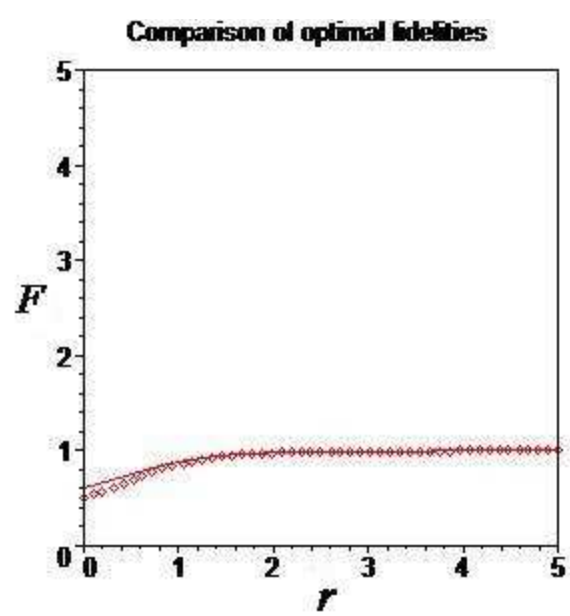


Fig.2